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Competitive evaluation based on integer-valued DEA model with different constraint sets[1]

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Abstract. To discuss the integer-valued DEA (data envelopment analysis) model, different nations' values different medals in different ways and the targets for the inefficient nations should be integer-valued in evaluating the performance of participating nations in the competition. It makes some adjustment to the model which considers different constraint sets to make the efficiency score of each nation lie between zero and unity. And it proposes the radial-based integer-valued DEA model (the RDI model) to obtain the integer-valued targets for the inefficient nations. Based on these, it presents an integer-valued DEA model with different constraint sets (the RDID model) to evaluate the participating nations at the competition. In addition, comparisons are made among the RDID model, the integer-valued DEA model with the same constraint set (the RDIS model) and the BCC model. The results demonstrated the feasibility and justice of the RDID model. Based on the above findings, it is concluded that the model can also be used to evaluate some similar problems in an instructive way.

Key words. Data envelopment analysis (DEA), different constraint sets, integer values, performance evaluating.

1. Introduction

Data envelopment analysis (DEA) has been widely used to evaluate the performance of the participating nations. DEA measures the relative efficiency of a set of decision making units (DMUs) through the programs, which started with the work of Charnes and Cooper. Later, more and more different cases led to the proposition of other models, such as the BCC, additive, hybrid, cross efficiency models, which broadened the application area of DEA. DEA possesses two notable advantages. One is that it does not have any assumptions on the production function, and the

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other is that it does not impose any subjective weights on the multiple inputs and outputs. Therefore, it can be widely used in evaluating the participating nations at the competition.

In some papers, two inputs (i.e., GNP and population) and three outputs (i.e., total numbers of golden, silver, bronze medals each participating nation got) are taken into consideration to appraise the performance of each nation based on a classical DEA model with restricted weights [1]. They found out that all the participating nations showed the positive or negative trends in the five consecutive Olympics Games. They ranked the efficient nations by just counting the times that the efficient nations appeared in the reference sets of the inefficient nations. However, the case that the two efficient nations appeared in the reference sets the same times might occur. A zero-sum game DEA model was proposed and used to evaluate the performance of participating countries, in which the two inputs (GDP and population) were considered and weights were also restricted. But they ignored the improvement of the inefficient nations. Although it was considered it, the linear combination of efficient nations in the referee sets may be the unattainable goals for the inefficient nations [2]. Some scholars used a two-stage method to analyze the achievements of participating nations by linking the self-organizing mappings to DEA model. Here the input indicators were GDP per capita, population, disability adjusted life expectancy and index of equality of child survival. They categorized the participating nations into several groups. But the combination of data mining and ranking based on DEA may not be perfect.

The technique of vote-ranking was combined with the cross-evaluation methods to assess the performance of the participating nations at the competition. It could rank the participating nations effectively, but it could not provide an efficient target for the inefficient nations owing to the ignorance of the difference between the inefficient nation and its frontier target. As a result, the targets provided the proper benchmarks for the inefficient ones. As for the problem, the Context-dependent DEA (CRA-DEA) model which allows multiple constraint sets have to be considered to be employed. But the efficiency scores may not always lie between zero and unity [3]. Also there does not exist a model which considers both the integer problem and different constraint sets for different DMUs.

This paper mainly tackles the above problems. In section 2, an integer-valued DEA model was proposed to evaluate the performance of participating nations while taking into consideration different constraint sets for different DMUs. In section 3, the model was used to evaluate the participating nations of the 2012 Olympics, the integer-valued targets were given for inefficient participating nations and they were ranked. In addition, some comparisons were made to show the justice and feasibility of the model.

2. Materials and methods

In this section, we introduce an integer-valued DEA model with different constraint sets under variable returns-to-scale. Supposing there are no decision making units (DMUs), each representing a participating nation.

GDP per capita is a measure for attainable resource to train athletes, build and maintain training facilities, develop better training methods and so on. Compared with GDP, GDP per capita is a better indicator to show the economic power of the nation [4]. So we use GDP per capita as one of the inputs instead of GDP. Population size determines the pool from which potential athletes can be drawn, so it is another significant indicator explaining Olympic achievement. GDP per capita and population are the most important indicators expressing the economic and demographic power of nations. Therefore, we use population and GDP per capita as the two inputs, and the number of golden, silver and bronzes medals as the three outputs.

2.1. DEA model with different constraint sets

Here we firstly introduce the output-oriented DEA model under variant returns-to-scale with the same constraint set. The model makes no exception to all the countries. If we use this model to evaluate the efficiency of the participating country, it seems somewhat improper. We need to refer to Cook and Zhu's CAR-DEA model to make Model 1 more suitable for the evaluation of all the participating nations at the competition.

Before introducing the DEA model with different constraint sets, we divide the participating nations of the competition into 4 groups according to the criteria from the World Bank. The first group includes the nations with the GDP below \$825 per capita. Under-developing countries from Africa and Middle-Asia belong to the group [5]. The second group contains the nations with GDP ranging from \$826 to \$3357 per capita. Some developing nations from Eastern Africa, Southern Africa and Eastern Europe are included in the group. The third group includes some developing and low developed countries from Middle-Europe, America and southern Africa with GDP ranging from \$3358 to \$10461 per capita [6]. The nations in the fourth group are well-developed, and most of them are from Western Europe and northern America with GDP above \$10462 per capita. Each group has its own constraint set in the form

$$c_{r\mathrm{L}}^{k}\mu_r \le \mu_l \le c_{r\mathrm{U}}^{k}\mu_r,\ k=1,2,3,4,\ r=2,3\,. \tag{1}$$

Here $c_{2\mathrm{L}}^{k}$ means that in the kth group, at least $c_{2\mathrm{L}}^{k}$ silver medals and not more than $c_{2\mathrm{U}}^{k}$ silver medals are equivalent to one golden medal. Symbols $c_{3\mathrm{L}}^{k}$ and $c_{3\mathrm{U}}^{k}$ can be interpreted in the same way. However, when we put all these different constraint sets for DMUs from different groups together, it may lead to unfeasible solution. As a result, some adjustments are made to tackle the problem. When $\mu_r^{'}=c_{r\mathrm{L}}^{k}/c_{r\mathrm{L}}^{l}\mu_r$, the restrictions can be replaced by $c_{r\mathrm{L}}^{l}\mu_r^{'}\le\mu_l\le(c_{r\mathrm{L}}^{l}/c_{r\mathrm{L}}^{k})c_{r\mathrm{U}}^{k}\mu_r^{'}$ [7].

The restrictions for different nations have the same lower bound in this way. As for the common upper bound, it can be determined by $\bar{c}_{r\mathrm{U}}=\min\left\{\bar{c}_{r\mathrm{U}}^{1},\ \bar{c}_{r\mathrm{U}}^{2},\ \bar{c}_{r\mathrm{U}}^{3},\ \bar{c}_{r\mathrm{U}}^{4}\right\}$ where $\bar{c}_{r\mathrm{U}}^{k}=\left(c_{r\mathrm{L}}^{1}/c_{r\mathrm{L}}^{k}\right)c_{r\mathrm{U}}^{k},\ k=1,2,3,4$.

According to the above adjustment, the output-oriented CAR-DEA model was obtained [8].

$$\begin{aligned}
&\text{Min} \sum_{i=1}^{2} v_i x_{ij} + \mu_0, \\
&\text{s.t.} \sum_{i=1}^{2} v_i x_{ij_k} + \mu_0 - \sum_{r=1}^{3} \mu_r \frac{c_{rL}^{l}}{c_{rL}^{k}} y_{rj_k} \geq 0, \ , k = 1, 2, 3, 4, \ j_k \in J_k, \\
&\sum_{r=1}^{3} \mu_r y_{rj} = 1, \\
&c_{r\mathrm{L}} \mu_r \leq \mu_l \leq c_{r\mathrm{U}} \mu_r, \ r = 2, 3, \\
&\mu_r, \ v_i \geq 0 \qquad \forall r, i, \mu_0 \text{free.}
\end{aligned} \tag{2}$$

Here x_{ij_k} denotes the ith input of DMU$_j j$ from the kth group and y_{ij_k} denotes the th output of the DMU. But the optimal values of Model 2 do not always exceed unity. Sometimes its optimal value is bigger than unity and sometimes it is smaller than unity. As a result, the efficiency scores which are the inverse of the optimal values do not always lie between zero and unity [9]. The reason for it is that we constrain $\sum_{r=1}^{s} \mu_r y_{r0} = 1$. As a matter of fact, the production frontier varies for DMUs from different group. In order to make the optimal value of the corresponding model bigger than unity, we have to make the weights of outputs consistent. As a result, we substitute $\sum_{r=1}^{s} \mu_r y_{r0} = 1$ with $\sum_{r=1}^{3} \mu_r \frac{c_{rL}^{l}}{c_{rL}^{k}} y_{rj(k0)}$ to tackle the problem. Here comes our revised model.

$$\begin{aligned}
&\text{Min} \sum_{i=1}^{2} v_i x_{ij} + \mu_0, \\
&\text{s.t.} \sum_{i=1}^{2} v_i x_{ij_k} + \mu_0 - \sum_{r=1}^{3} \mu_r \frac{c_{rL}^{l}}{c_{rL}^{k}} y_{rj_k} \geq 0, \ , k = 1, 2, 3, 4, \ j_k \in J_k, \\
&\sum_{r=1}^{3} \mu_r \frac{c_{rL}^{l}}{c_{rL}^{k0}} y_{rj_{k0}} = 1, \\
&\bar{c}_{r\mathrm{L}} \mu_r \leq \mu_l \leq \bar{c}_{r\mathrm{U}} \mu_r, \ r = 2, 3, \\
&\mu_r, \ v_i \geq 0 \qquad \forall r, i, \mu_0 \text{free.}
\end{aligned} \tag{3}$$

As for the determination of weights, such as, and so on, we have to conform to the classification of the nations. Then we can get Table 1 to show the range of each weight.

According to Table 1, we find out that countries in the fourth group value the golden medals most. In their eyes, at least 3 silver medals or at least 4 bronzes medals are equivalent to one golden medal. No more than 5 silver medals or 8 bronze medals are equivalent to one golden medal [10]. But nations in the first group even regard one silver medal or one bronze medal equivalent to one golden medal, which shows these nations do not care about whether the medals they obtain are golden or not. In their mind, any achievements at the competition are their pride. All these can clearly demonstrate the different attitudes of different nations to the medals, which match the development situation of each nation well. As for the common ratio, we can obtain them based on the above adjustment scheme. Therefore, we can insert the ratio into Model 3. In order to elaborate the model, we can get its dual form.

Table 2. Inputs and outputs of participating nations of 2012 Olympics

Ratio	Bound	Group 1: $k=1$	Group 2: $k=2$	Group 3: $k=3$	Group 4: $k=4$	Common ratio
	Lower	1	1	2	3	1
	Upper	2	2	4	5	1.6667
	Lower	1	1	3	4	1
	Upper	2	2	6	8	2

2.2. Integer-valued DEA model

In the above models, the targets are not necessarily the whole numbers. Therefore, when the targets are not the whole numbers, they cannot work as the benchmarks for the inefficient DMU to improve its performance. Just rounding the number to the nearest whole number may lead to the overestimation or underestimation [11]. It does not make any difference to large nations, while it makes much difference to small nations. In order to tackle the problem, we propose an integer-valued DEA model: radial distance-based integer-valued DEA model (hereafter referred to as RDI model). Firstly, we introduce it in its input-oriented form.

We can elaborate it through Fig. 1.

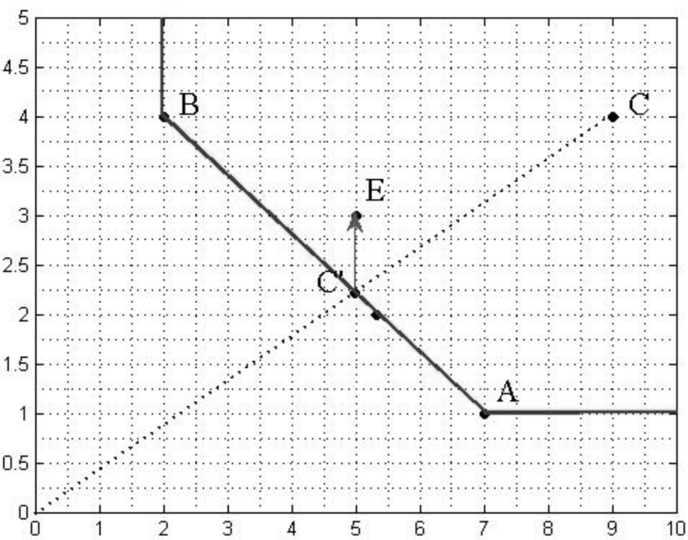

Fig. 1. Ways to find the optimal value based on the RDI model

It is noted that in the objective function of the RDI model, the objective of the slacks is searching for its minimal norm. The reason for it is that the smaller is, the closer to the production frontier the targets are. In the RDI model, the optimal is in the R+ direction of. In other words, in the RDI model, we first determine the point, and then search for based on (the green path in Fig. 1) [12]. Obviously, the optimal solution of is always obtained in the intersection of PPS and the radial direction line, which means the optimal obtained in the RDI model is the same as the one got in the CCR model.

In Fig. 1, C is the DMU under estimation. We can also describe the process of searching for the optimal point of the RDI model in two steps. First, we search for the intersection of PPS and the radial direction line (i.e., point C' in Fig. 1). Second, we start from the intersection point, search along the R+ direction for a certain integer-valued point with the minimal norm (i.e., point E in Figure 1), because any point in the R+ direction of the intersection point with a larger norm is much worse.

3. Results

We categorize all the participating nations of the 2012 London Olympics into four groups based on the criteria from the World Bank mentioned above. We use population and GDP per capita as the two inputs, and the number of golden, silver and bronzes medals as the three outputs.

Table 2. Inputs and outputs of participating nations of 2012 Olympics

Nation	GDP per capita (dollar)	Population	Golden medals	Silver medals	Bronze medals	Group
America	51601.37	313232000	46	29	29	4
Britain	41706.77	62698360	29	17	19	4
Russia	14541.39	138739900	24	25	33	4
South Korea	25080.82	48754660	13	8	7	4
Germany	43367.65	61471830	11	19	14	4
France	41136.59	65312250	11	11	12	4
Italy	34281.72	61016800	8	9	11	4
Hungary	12712.91	9976062	8	4	5	4
Australia	70494.16	21766710	7	16	12	4
Japan	47080.01	126475700	7	14	17	4
Republic of Kazakhstan	13111.22	15522370	7	1	5	4
Netherlands	48859.66	16847010	6	6	8	4
New Zealand	39964.46	4290347	5	3	5	4

We collect GDP per capita and population of each participating nation at the 2012 Olympics from the official website of the World Bank. The medals each participating nation obtained are gathered from the official website of the Olympics [13]. We can obtain Table 2.

There are six nations in the first group, seven nations in the second group, twenty-four nations in the third group and forty-eight nations in the fourth group. Therefore, $J1$, $J2$, $J3$ and $J4$ represent 6, 7, 24 and 48, respectively [14]. Then we can evaluate the participating nations at the 2012 Olympics based on the RDID model.

Table 3. Efficiency score and targets of each participating nation based on RDID model

Nation	η	Efficiency score	Target for golden medals	Target for silver medals	Target for bronze medals	Group
America	1	1	46	29	29	4
Britain	1	1	29	17	19	4
Russia	1	1	24	25	33	4
South Korea	1.47	0.69799	17	11	14	4
Germany	1.57	0.638127	28	18	22	4
France	1.92	0.519108	29	17	19	4
Italy	2.06	0.484438	24	15	18	4
Hungary	1	1	8	4	5	4
Australia	1	1	7	16	12	4
Japan	2.08	0.480042	25	23	30	4
Republic of Kazakhstan	1.39	0.719425	8	5	6	4
Netherlands	1.28	0.780009	8	9	8	4
New Zealand	1	1	5	3	5	4

The optimal values of all participating nations lie above unity. The efficiency scores are the inverse of the optimal scores and they all lie between zero and unity. The participating nations with the high efficiency score behave well at the 2012 Olympics. The targets of participating nations are all the whole numbers, which can be exactly used as the improvement benchmarks for the inefficient participating nations. The efficiency scores of the RDID model and targets of each nation are listed in the Table 4.

In the first group, the nations are under-developed. The GDP per capita of each nation in the group is the lowest in the four groups. They do not value the golden medals too much. There is only one nation whose efficiency score is unity, which denotes that there is no improvement for the nation. So its targets are the same as the medals it got. The efficiency score of Afghanistan is the lowest, which means there is much improvement for Afghanistan. Its targets are 4, 4 and 4, so it needs to get four more golden medals, four more silver medals and three more silver medals to be efficient.

In the second group, there are no efficient participating nations. The efficiency score of Kenya is the highest and it only needs to get another two golden medals to be efficient based on the medals it obtained. Among the participating nations in the second group, the targets of Morocco are the highest and its medals is the lowest. Therefore, its efficiency score is the lowest.

In the third group, there are four efficient nations, namely China, Jamaica, Ukraine and Grenada. The efficiency score of Turkey is the lowest in the group and it needs to acquire eleven more golden medals to become efficient.

In the fourth group, there are seven efficient nations in the group, namely America, Britain, Russia, Hungary, Australia, New Zealand and Panama. The efficiency score of Argentina is the lowest, and it need to get another eleven golden medals, eight silver medals and nine bronze medals to become efficient. The nations in the group value the golden medals more than three other groups, which can be seen from the AR ranges in Table 1.

Next, we make a comparison between the efficiency scores of the RDID model and the RDI model with the same constraint set (hereafter referred to as the RDIS model). As for the RDIS model, we set and in order to make a clear comparison. And we rank the nations according to the optimal value in the ascending order. As for efficient nations, they are all ranked as the first placers. Nations in the first and second groups are underestimated through the RDID model, compared to the RDIS model. Because in the RDID model, the golden medals of these nations are considered less important than these in the RDIS model [15]. Nations in the third and fourth groups are overestimated through the RDID model, compared to the RDIS model. The reason for it is that the golden medals of these nations are considered more important in the RDID model than these in the RDIS model.

At last, we make a comparison between the targets we obtain and the targets of the BCC model. We find out that the targets we obtain through the RDID model are all the integer-valued, which can work as the benchmarks for the inefficient nations. But the targets through the BCC model are generally fractional, which cannot work as the targets of the inefficient nation effectively. We also find out that the targets provided by the RDID model are not always a rounding up or down of the fractional targets. For example, the targets of South Korea through the BCC model are (21.09, 12.98, 13.75), but its targets through the RDID model are (17, 11, 14). Even if the targets obtained by the BCC model are the whole numbers, they are worse than the targets of the RDID model, such as Azerbaijan. Its targets through the RDID model are (5, 4, 5), while its targets through the BCC model are (2, 2, 6). It is obvious that (5, 4, 5) are better than (2, 2, 6). The comparisons above show that the RDID model not only provide a proper integer-valued target for the inefficient nations but also evaluate the participating nations from different views.

Table 4. Comparison between the RDID and RDIS models

Nations	RDID	Ranking	RDIS	Ranking
America	1	1	1	1
Britain	1	1	1	1
Russia	1	1	1.25	6
South Korea	1.47	9	1.68	13
Germany	1.57	10	1.71	14
France	1.92	17	2.17	21
Italy	2.06	20	2.51	27
Hungary	1	1	1.000944	2
Australia	1	1	1	1
Japan	2.08	22	2.93	29
Republic of Kazakhstan	1.39	8	1.64	12
Netherlands	1.28	6	1.37	9
New Zealand	1	1	1.005449	3

4. Conclusion

The RDID model is mainly focused on to evaluate the participating nations at the Olympics. There are two main priorities for the model. Firstly, it can provide a more reasonable target for the inefficient nations because its targets are the whole numbers. People do not need to round the targets to the nearest whole number to get the integer-valued targets. It can provide proper targets for the inefficient nations. Secondly, it considers the different views of different participating nations to golden, silver and bronze medals and makes some adjustment to the original model to make the efficiency score lie between zero and unity. Moreover, the participating nations are divided into four group based on the criteria of the World Bank and give the proper constraint set to each group. Therefore, the RDID model can evaluate the participating nations from different view. And the comparisons between the RDID and RDIS models are made, and the RDID and BCC models again demonstrate the feasibility and justice of the RDID model. In conclusion, the model is instructive because the similar problems can be tackled through it.

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